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Thin Film Modeling of Delamination Buckling in Pressure Loaded Laminated Cylindrical Shells

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Delamination is one of the basic defects inherent to laminated shell structures. Under uniform external pressure such delaminations may buckle and subsequently propagate. This phenomenon is modeled here as a first approximation by considering a two-dimensional geometry (ring approximation) and a thin delaminated layer. Growth is studied by a fracture mechanics-based energy release rate criterion. Closed-form expressions for the critical pressure and growth conditions are derived, as well as for the cutoff level of the delamination range below which local delamination buckling cannot take place. A formulation that accounts for the effects of transverse shearing forces is also presented.

Introduction

A CLASS of important structural applications of fiber-reinforced composite materials involves the configuration of laminated shells. Although thin plate construction has been the thrust of the initial applications, much attention is now being paid to configurations classified as moderately thick shell structures. Such designs can be used in components in the aircraft and automobile industries, as well as in the marine industry (e.g., composite hulls for submersibles). Moreover, composite laminates have been considered in space vehicles in the form of circular cylindrical shells as a primary load-carrying structure. Besides composite structures, this problem could be of more general interest, as related to the adherence of preventive surface coatings, for example, to enhance corrosion and wear resistance.

The study of delamination behavior is needed because when the application of layered composites to engineering structures is contemplated, it is essential to answer not only the fundamental questions regarding the strength and stiffness of the material but also the question of damage tolerance, i.e., the ability of the system to resist failure in the presence of defects. The manufacture of composites requires involved procedures that may potentially result in the existence of defects in the finished product.[1] Indeed, local spalling or debonding may occur due to manufacturing imperfections or due to service loads that may include moisture-induced stresses and deformations, impact, and vibrations. As a consequence, structural elements with delaminations under compressive stress fields suffer a degradation of their stiffness and buckling strength and potential loss of integrity from possible growth of the interlayer crack.

Delamination buckling in plates under compression has received considerable attention and numerous contributions have addressed related issues in both one-dimensional and two-dimensional treatments.[2-6] However, delaminated shells have not yet been adequately investigated. Very few investigations have been carried out in this area. The buckling of stiffened circular cylindrical shells, with two unbonded orthotropic layers, is reported in Ref. 7. In this work it was assumed that the layers do not separate during buckling, i.e., the deformation of both layers was assumed to be the same. The case when one of the two unbonded orthotropic layers is circumferentially cracked was also examined. The results in Ref. 7 for a cylindrical shell made of aluminum with ablative outer layer and subjected to hydrostatic pressure show that the ablative layer had to be increased by 50% in thickness in the damaged (debonded) cylindrical shell to obtain the same buckling load as that of the perfect cylindrical shell. In Ref. 8 the effect of longitudinal delamination in a thin laminated cylindrical shell on the critical external pressure was examined, but this study presented only the governing equations with some numerical results for thin shells and did not provide closed-form solutions nor did it consider the problem of growth of the delamination. Likewise, very interesting numerical results were presented in Ref. 9 for the instability of symmetric cross-ply delaminated thin cylindrical shells.

One of the cases of delamination buckling is the bifurcation (adjacent-equilibrium) mode, which occurs with delaminations near the outside surface of the shell. The other case, which involves the configuration of a local debond near the inside surface of the shell, differs fundamentally from the usual case of beam or plate buckling under compression where the equilibrium form after buckling is close to the equilibrium form before buckling. This is due to the inherent curvature in the shell geometry that makes the transition to a new equilibrium state occur by a snap and makes the new equilibrium form to differ essentially from the initial one. On this subject of snap through delamination buckling there has been a recent study in the case of plates under pure bending (snap through buckling occurs because of the induced initial bending deformation).[10]

The thin film model approximation was first introduced by Chai et al.[2] for rectilinear plate geometries. Such first approximation analyses are useful because the results are quite simple and illustrative of the results for the more complete models. In this problem, besides the two-dimensional approximation (ring geometry), an additional approximation is made regarding the thickness of the delaminated layer, i.e., the unbuckled portion of the shell has been considered "infinitely thick" with respect to the delaminated layer. The closed-form expressions derived allow a direct assessment of the effect of location of the delamination through the thickness and its size on the buckling characteristics.

Analysis

In the present study, the configuration under study is represented in Fig. 1 and consists of a homogeneous, orthotropic ring of thickness h_s and of unit width containing a single delamination at depth h_l from the top surface. The configuration is subjected to a uniform pressure load p. The delamination is symmetrically located over the range $-\theta_0<\theta<\theta_0$. Over this region, the structure consists of the part above the delam-

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ination, of thickness h_I, referred to as part I, and the part below the delamination, of thickness h_{II}, referred to as part II. The remaining part of the structure that is intact and of thickness h_s is referred to as the "base shell" and the subscript s is used. The latter is extended over the range $-(\pi - \theta_0) < \theta_s < (\pi - \theta_0)$. The line $\theta = 0$ is the same for parts I and II but is diametrically opposite for the "base shell" (Fig. 1).

The laminate is loaded by a uniform pressure p that in turn creates a compressive hoop stress field; at the critical level we assume that local buckling of the delaminated layer only (part I) occurs, the lower part II and the base shell undergoing no additional θ-dependent radial displacements, except possibly for a uniform contraction.

Denote by $w_i(\theta)$ the radial and by $v_i(\theta)$ the circumferential displacements of the midsurface of each part. The corresponding resultant internal forces and moments are denoted by $T^i_{\theta\theta}(\theta)$ and $M^i_{\theta\theta}(\theta)$, respectively. In the following, R denotes the midsurface radius of curvature, taken the same for all three parts. An additional quantity that will be used is the midsurface rotation β_i defined by

$$\beta_i = (1/R)(v_i - w_{i,\theta}) \tag{1a}$$

The prebuckling state of uniform external compression p is characterized by the displacement field:

$$v_i^0(\theta) = 0, \quad w_i^0(\theta) = w_0 = -\frac{R^2 p(1-\nu_{12}\nu_{21})}{E_2 h_s}$$

$$\beta_i^0(\theta) = 0, \quad i = I, II, s \tag{1b}$$

In denoting the material properties we have used the subscripts 1 for the radial direction and 2 for the circumferential direction; hence, E_2 is the modulus of elasticity along the circumferential (normally reinforcing) direction. There is also a prebuckling internal resultant force and moment field:

$$T^{i0}_{\theta\theta}(\theta) = -pR(h_i/h_s), \quad M^{i0}_{\theta\theta}(\theta) = 0, \quad i = I, II, s \tag{1c}$$

The buckled shape of the film is now represented by the displacement field, where the superscript a represents the additional (to the prebuckled state) quantities:

$$w_I^a(\theta) = A\cos(\pi/\theta_0)\theta, \qquad v_I^a(\theta) = B\sin(\pi/\theta_0)\theta \tag{2a}$$

For the other parts

$$w_{II}^a(\theta) = w_s^a(\theta) = -A, \qquad v_{II}^a(\theta) = v_s^a(\theta) = 0 \tag{2b}$$

Notice that this displacement field satisfies the kinematic boundary conditions at the interface since all three parts have the same displacements and rotations, i.e., $\beta_i(\theta_0) = 0$, $w_i(\theta_0) = -A$, and $v_i(\theta_0) = 0$, for $i = I, II, s$.

The nonlinear differential equations of equilibrium (nonlinear Donnell shell theory) to be satisfied for each part are[11]

$$RT^i_{\theta\theta,\theta} + M^i_{\theta\theta,\theta} = 0 \tag{3a}$$

$$M^i_{\theta\theta,\theta\theta} - RT^i_{\theta\theta} - RT^i_{\theta\theta}\beta_{i,\theta} = p_i R^2 \tag{3b}$$

where $p_i = p$ for part I and for the base shell, and $p_i = 0$ for part II.

In the previous equations, the resultant forces and moments for each part can be found from the displacement field as follows:

$$T^i_{\theta\theta} = \frac{E_2 h_i}{(1-\nu_{12}\nu_{21})R}(w_i + v_{i,\theta})$$

$$M^i_{\theta\theta} = \frac{E_2 h_i^3}{12(1-\nu_{12}\nu_{21})R^2}(v_{i,\theta} - w_{i,\theta\theta}) \tag{4}$$

The linear form of the resultant force/moment-displacement equations is used since, in linear stability analysis, terms quadratic or cubic in w_i^a and v_i^a are omitted because of the smallness of the incremental displacements.

The displacements in the previous relations are both the prebucking and buckling, i.e.,

$$w_i = w_i^0 + w_i^a, \qquad v_i = v_i^0 + v_i^a \tag{5a}$$

Since the postbuckled shape is a perturbation of the prebuckling state, the additional quantities are of first order (can be infinitesimally close to the initial state). Therefore, substituting the expressions for the resultant force and moment into the differential equations of equilibrium and retaining first-order terms, we obtain

$$w^a_{,\theta} + v^a_{,\theta\theta} + \frac{h_i^2}{12R^2}(v^a_{,\theta\theta} - w^a_{,\theta\theta\theta}) = 0 \tag{5b}$$

$$\frac{h_i^2}{12R^2}(v^a_{,\theta\theta\theta} - w^a_{,\theta\theta\theta\theta}) - (w^a + v^a_{,\theta}) - \frac{w_0}{R}(v^a_{,\theta} - w^a_{,\theta\theta}) = 0 \tag{5c}$$

Using the expressions for the displacement field of Eq. (2a) in the first of the previous relations gives for the buckled layer

$$A = -B\left(1 + \frac{h_I^2}{12R^2}\right)\frac{\pi}{\theta_0}\Bigg/\left(1 + \frac{h_I^2}{12R^2}\frac{\pi^2}{\theta_0^2}\right) \tag{6}$$

Finally, substituting Eqs. (2a) and (6) into the second differential equation, Eq. (5c), gives the critical conditions as follows:

$$\frac{w_{0cr}}{R} = \frac{h_I^2}{12R^2}\left(1 - \frac{\pi^2}{\theta_0^2}\right) \tag{7a}$$

or, from Eq. (1b), the critical pressure

$$p_{cr} = \frac{E_2}{12(1-\nu_{12}\nu_{21})}\frac{h_I^2 h_s}{R^3}\left(\frac{\pi^2}{\theta_0^2} - 1\right) \tag{7b}$$

In the previous analysis, the Donnel shell theory equations for dead loading were used. If the equations for fluid-pressure loading are employed,[11] the last term in Eq. (5c) would be $w_0(w^a + w^a_{,\theta\theta})/R$ instead of $-w_0(v^a_{,\theta} - w^a_{,\theta\theta})/R$. Following the same steps as before would lead to the critical pressure

$$p_{cr} = \frac{E_2}{12(1-\nu_{12}\nu_{21})}\frac{h_I^2 h_s}{R^3}\left(\frac{\pi^2}{\theta_0^2} - 1\right)\Bigg/\left(1 + \frac{h_I^2}{12R^2}\right) \tag{7c}$$

i.e., a smaller value by the factor $1/[1 + (h_I^2/12R^2)]$. The correction is insignificant for thin delaminations ($h_I \ll R, h_s$).

Now let us consider the question of delamination growth. As long as the load or, alternatively, the compressive hoop strain in the base shell ϵ is below the critical value, $\epsilon_{cr} = w_{0cr}/$

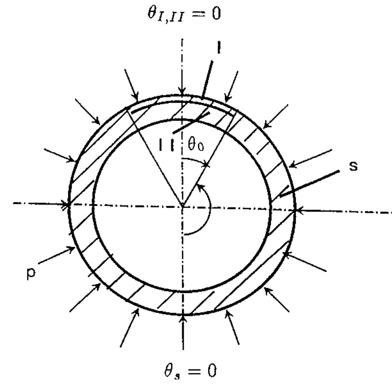

Fig. 1 Definition of the geometry. A delamination size $\pm\theta_0$ exists at a depth h_I from the outside surface.

r, the delamination does not lose stability. For $\epsilon > \epsilon_{cr}$ the delamination buckles, and the buckling mode is given by Eq. (2a) in terms of the constant A that represents the buckling level and can be determined from compatibility requirements in terms of the compressive hoop strain in the base shell ϵ and the critical one ϵ_{cr} since $\epsilon = \epsilon_{cr} + A/R$. In this way the energy release rate $G = G(\theta_0, \epsilon_{cr})$ is a function of the compressive strain in the shell, and G can be plotted as a family of curves for each value of the strain level (or alternatively A). A similar analysis was employed in Ref. 6 for delaminations in plates. The complete postbuckling solution taking geometric nonlinearities into account (such as in Ref. 4 for a plate configuration) would provide a more accurate description of delamination growth. Pending such a solution, we use the present buckling analysis to obtain insight into the growth problem in the context of the present approximate formulation. Furthermore, if the eigenmode from the buckling solution (zeroth approximation) is used to start the iteration cycle in Newton's method of computing numerical solutions for the nonlinear governing equations,[12] it has been shown[13] that the zeroth approximation for the postbuckling pressure-deflection curves can compare well with the converged Newton's method.

Define a reduced thickness quantity:

$$t_i = \frac{h_i^2}{12R^2}, \qquad i = I, II, s \tag{8}$$

The strain energy in the buckled layer consists of the extensional energy (due to $T_{\theta\theta}$) and the bending energy (due to $M_{\theta\theta}$) and is given by

$$U_I = \frac{(1-\nu_{12}\nu_{21})}{2Eh_I}\int_{-\theta_0}^{\theta_0} T_{\theta\theta}^2(\theta) R_I \, d\theta + \frac{12(1-\nu_{12}\nu_{21})}{2Eh_I^3}\int_{-\theta_0}^{\theta_0} M_{\theta\theta}^2(\theta) R_I \, d\theta \tag{9a}$$

Using Eqs. (6), (4), (7a), and (2a) results in

$$U_I = \frac{Eh_I}{2(1-\nu_{12}\nu_{21})R} A^2 \frac{t_I}{1+t_I}\left(1 - \frac{\pi^2}{\theta_0^2}\right)^2 \theta_0 \tag{9b}$$

Since for parts II and s,

$$T_{\theta\theta}^{IIa} = -A\frac{E_2 h_{II}}{(1-\nu_{12}\nu_{21})R}, \qquad M_{\theta\theta}^{II} = 0 \tag{10a}$$

$$T_{\theta\theta}^{sa} = -A\frac{E_2 h_s}{(1-\nu_{12}\nu_{21})R}, \qquad M_{\theta\theta}^{s} = 0 \tag{10b}$$

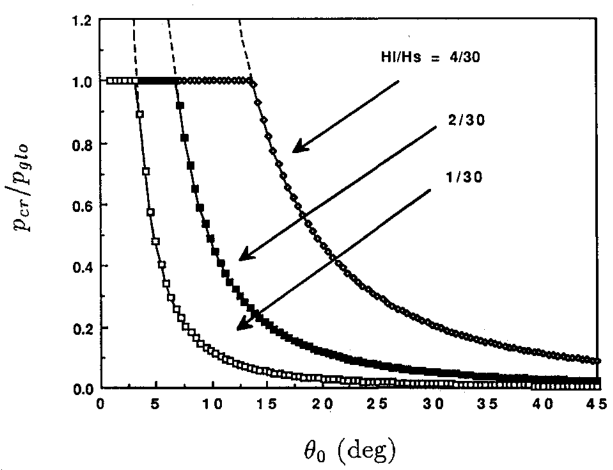

Fig. 2 Delamination buckling pressure vs delamination size θ_0 for several values of delamination depth h_I; p_{glo} is the global buckling load for the entire shell.

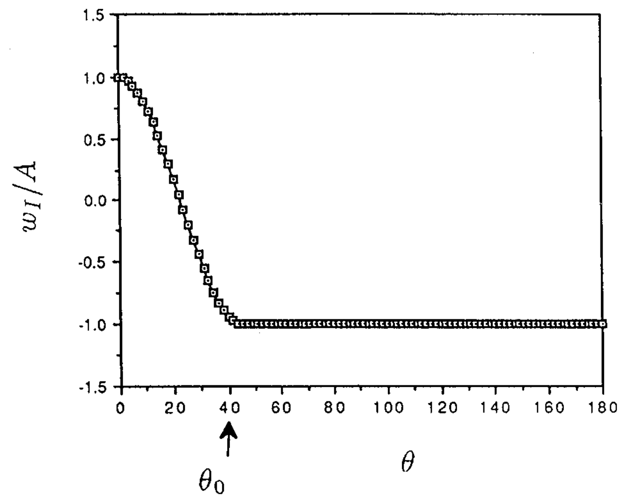

Fig. 3 Form of stability loss by local delamination buckling.

we obtain

$$U_{II} = \frac{E_2 h_{II}}{(1-\nu_{12}\nu_{21})R} A^2 \theta_0, \qquad U_s = \frac{E_2 h_s}{(1-\nu_{12}\nu_{21})R} A^2 (\pi - \theta_0) \tag{11}$$

The potential energy Π is

$$\Pi(\theta_0) = U_I + U_{II} + U_s - W_E \tag{12a}$$

where W_E is the work of the external load. Since the load remains constant, $W_E = 2(U_I + U_{II} + U_s)$ and

$$\Pi = -(U_I + U_{II} + U_s) \tag{12b}$$

Since the incremental delamination growth is $R\, d\theta_0$, the energy release rate is

$$G = -\frac{1}{R}\frac{d\Pi}{d\theta_0} \tag{13}$$

Taking into account $h_{II} = h_s - h_I$ and using Eqs. (9–13), we obtain the energy release rate

$$G = \frac{Eh_I}{2(1-\nu_{12}\nu_{21})R^2} A^2 \left[\frac{t_I}{1+t_I}\left(\frac{\pi^2}{\theta_0^2} - 1\right)\left(1 + 3\frac{\pi^2}{\theta_0^2}\right) + 2\right] \tag{14}$$

If we adopt the growth criterion of a critical level of the energy release rate G_c, then the previous equation can be used to assess the growth conditions for a given delamination range θ_0 in terms of the midpoint deflection of the buckled layer A or, equivalently, the additional compressive hoop strain in the base shell, $\epsilon_{\theta\theta,s}^a = A/R$.

Transverse Shear Effects

In studying stability problems of composite materials, consideration should be given to the effect of the transverse shearing force that is introduced in the process of buckling deflections. This is because of the relative low ratio of shear to extensional modulus of composites as opposed to that of their metal counterparts. Transverse shear effects in delamination buckling of composite plates have been considerd in Ref. 3.

An accurate description of transverse shear effect would involve using higher order shell theories.[14,15] For the present study we employ a first-order shear deformation theory. The displacement field throughout the delaminated layer $W(r,\theta)$ and $V(r,\theta)$ is described in terms of the midsurface displacements $w(\theta)$ and $v(\theta)$ and the additional rotation function $\psi(\theta)$ so that

$$W(r,\theta) = w(\theta), \qquad V(r,\theta) = v(\theta) + \eta\psi(\theta) \tag{15}$$

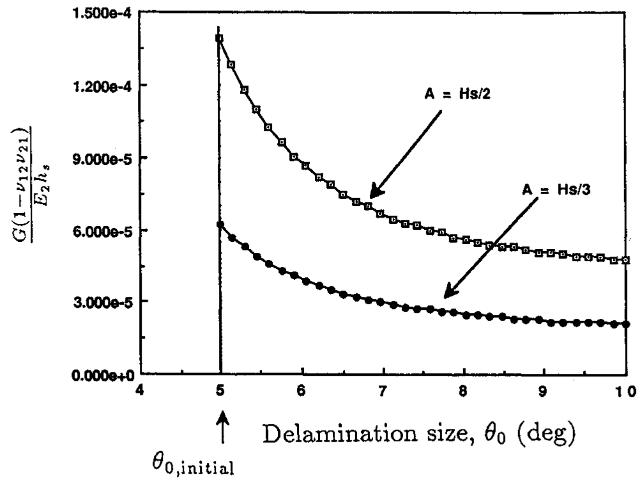

Fig. 4 Energy release rate vs delamination size θ_0 for several values of midpoint deflection of the delaminated layer. An initial delamination size of $\theta_0 = 5^0$ and a thickness of $h_I/h_s = 2/30$ were assumed.

where η is the thicknesswise coordinate from the midsurface. Using the nonlinear strain definitions in the expression for the strain energy and the principle of stationary total potential allows the derivation of the governing equations for the nonlienar case under consideration in the same fashion as for the usual linear case.[15]

Three nonlinear differential equations of equilibrium, to be satisfied for the delaminated layer, are found by this approach:

$$T_{\theta\theta,\theta} + T_{r\theta} = 0 \tag{16a}$$

$$T_{\theta\theta} + RN_{\theta\theta}\beta_{,\theta} - T_{r\theta,\theta} = -pR \tag{16b}$$

$$M_{\theta\theta,\theta} - RT_{r\theta} = 0 \tag{16c}$$

The resultant forces and moments in the previous relations are defined as

$$T_{\theta\theta} = \int_{-h/2}^{h/2} \sigma_{\theta\theta}\, \mathrm{d}\eta, \quad N_{\theta\theta} = \int_{-h/2}^{h/2} \frac{\sigma_{\theta\theta}}{R+\eta}\, \mathrm{d}\eta \tag{17a}$$

$$M_{\theta\theta} = \int_{-h/2}^{h/2} \sigma_{\theta\theta}\eta\, \mathrm{d}\eta, \quad T_{r\theta} = \int_{-h/2}^{h/2} \tau_{r\theta}\, \mathrm{d}\eta \tag{17b}$$

Therefore, in this case the resultant forces and moments can be found from the displacement field as follows:

$$T^i_{\theta\theta} = \frac{E_2}{(1-\nu_{12}\nu_{21})}\left[\ell n\frac{R+h_i/2}{R-h_i/2}(w_i + v_{i,\theta} - R\psi_{,\theta}) + h_i\psi_{,\theta}\right] \tag{18a}$$

$$M^i_{\theta\theta} = \frac{E_2}{(1-\nu_{12}\nu_{21})}\left(h_i - R\,\ell n\frac{R+h_i/2}{R-h_i/2}\right)(w_i + v_{i,\theta} - R\psi_{,\theta}) \tag{18b}$$

$$N^i_{\theta\theta} = \frac{E_2}{(1-\nu_{12}\nu_{21})}\left[\frac{h_i}{R^2 - h_i^2/4}(w_i + v_{i,\theta} - R\psi_{,\theta}) + \ell n\frac{R+h_i/2}{R-h_i/2}\psi_{,\theta}\right] \tag{18c}$$

$$T^i_{r\theta} = G_{12}\,\ell n\frac{R+h_i/2}{R-h_i/2}(w_{i,\theta} - v_i + R\psi) \tag{18d}$$

The classical shell theory corresponds to $R\psi(\theta) = -(w_{,\theta} - v + w_{0,\theta}) = R\beta$. In general, the previous equations become the classical shell theory, Eqs. (3) and (4), if we substitute $N_{\theta\theta}$ with $T_{\theta\theta}/R$.

In the prebuckling state

$$\psi_0 = 0, \qquad N^0_{\theta\theta} = \frac{E_2}{(1-\nu_{12}\nu_{21})}\frac{h_I}{R^2 - h_I^2/4}w_0 \tag{19}$$

Using the same form for $w(\theta)$ and $v(\theta)$ as in Eq. (2a) and setting

$$\psi = \psi_0 + \psi^a, \qquad \psi^a(\theta) = C\sin(\pi/\theta_0)\theta \tag{20}$$

we obtain from Eqs. (16) and (18) the governing equations for the first-order quantities as follows:

$$w^a_{,\theta} + v^a_{,\theta\theta} - R\psi^a_{,\theta\theta} + (h_I/q)\psi^a_{,\theta\theta} + \frac{G_{12}(1-\nu_{12}\nu_{21})}{E}(w^a_{,\theta} - v^a + R\psi^a) = 0 \tag{21a}$$

$$w^a + v^a_{,\theta} - R\psi^a_{,\theta} + \frac{h_I}{q}\psi^a_{,\theta} - \frac{G_{12}(1-\nu_{12}\nu_{21})}{E}(w^a_{,\theta\theta} - v^a_{,\theta} + R\psi^a_{,\theta}) + \frac{N^0_{\theta\theta}(1-\nu_{12}\nu_{21})}{Eq}(v^a_{,\theta} - w^a_{,\theta\theta}) = 0 \tag{21b}$$

$$\frac{E}{G_{12}(1-\nu_{12}\nu_{21})}\left(\frac{h_I}{qR} - 1\right)(w^a_{,\theta} + v^a_{,\theta\theta} - R\psi^a_{,\theta\theta}) - (w^a_{,\theta} - v^a + R\psi^a) = 0 \tag{21c}$$

where

$$q = \ell n\frac{R + h_I/2}{R - h_I/2} \tag{22a}$$

Set

$$\tilde{N}^0_{\theta\theta} = \frac{N^0_{\theta\theta}}{G_{12}q}\frac{\pi^2}{(\pi^2 - \theta_0^2)} \tag{22b}$$

The first and third equilibrium equations, Eqs. (21a) and (21c), give

$$(B + Ak)(1 + \tilde{N}^0_{\theta\theta}) = CR \tag{23a}$$

$$(A + Bk) = CR\frac{\pi}{\theta_0}\left[\frac{G_{12}(1-\nu_{12}\nu_{21})\theta_0^2}{E_2\pi^2}\frac{\tilde{N}^0_{\theta\theta}}{(1+\tilde{N}^0_{\theta\theta})} - \frac{h_I}{qR} + 1\right] \tag{23b}$$

Substituting in the second equilibrium equation, Eq. (21b), gives the critical load from

$$N^0_{\theta\theta} = \frac{E_2}{(1-\nu_{12}\nu_{21})R}\left(\frac{\pi^2}{\theta_0^2} - 1\right) \times \frac{(h_I - Rq)}{\{1 - [E_2\pi^2/G_{12}(1-\nu_{12}\nu_{21})\theta_0^2 Rq](h_I - Rq)\}} \tag{24a}$$

Hence, from Eqs. (19) and (1b), the critical pressure including transverse shear effects is

$$p_{cr} = -\frac{E_2 h_s}{(1-\nu_{12}\nu_{21})Rh_I}\left(\frac{\pi^2}{\theta_0^2} - 1\right)\left(1 - \frac{h_I^2}{4R^2}\right) \times \frac{(h_I - Rq)}{\{1 - [E_2\pi^2/G_{12}(1-\nu_{12}\nu_{21})\theta_0^2 Rq](h_I - Rq)\}} \tag{24b}$$

Discussion of Results

Depending on the delamination size and location through the thickness, local buckling of the delaminated layer may not always occur before buckling of the entire shell. The critical pressure for buckling of the entire structure (global buckling pressure, p_{glo}) is given by[16]

$$p_{\mathrm{glo}} = \frac{E_2 h_s^3}{4(1-\nu_{12}\nu_{21})R^3} \tag{25}$$

This may well be below the critical pressure for local delamination buckling [Eq. (7b)].

This concept is illustrated in Fig. 2, which shows the critical buckling pressure vs delamination range θ_0 for various locations of the delamination. For $h_l/h_s = 1/30$, local delamination buckling cannot occur for θ_0 less than about 3.5 deg; for $h_l/h_s = 2/30$, local delamination buckling does not take place for θ_0 less than about 7 deg. For $h_l/h_s = 4/30$, the cutoff range is increased to 13.5 deg.

For delamination ranges beyond this point, the critical pressure is strongly dependent on the size of the delamination, rapidly dropping with increasing θ_0. By equating Eqs. (25) and (7b) we can find the cutoff angle θ_{0c} below which delamination buckling does not occur:

$$\theta_{0c} = \frac{\pi h_l}{\sqrt{3h_s^2 + h_l^2}} \tag{26}$$

In producing the results of Fig. 2, we have assumed a composite circular cylindrical shell of thickness over mean radius ratio $h_s/R_0 = 0.05$. It is supposed to be made, for example, by filament winding, with the fibers oriented around the circumference. The moduli in GN/m^2 and Poisson's ratios used (typical for a glass/epoxy material) are listed next, where 1 is the radial (r) and 2 is the circumferential (θ) direction: $E_2 = 57.0$, $\nu_{12} = 0.068$, $\nu_{21} = 0.277$. A uniform pressure p is applied at the outside boundary. A delamination of angular range θ_0 exists at a depth h_l from the outside surface.

Figure 2 also shows the effect of the location of the delamination through the thickness. For the same delamination range θ_0 the critical load decreases for delaminations located closer to the outer surface (i.e., smaller h_l/h_s). For $h_l/h_s = 2/30$, the critical buckling pressure is four times the one for $h_l/h_s = 1/30$ for a delamination of $\theta_0 = 10$ deg.

Figure 3 shows the buckled configuration corresponding to the assumption of Eq. (2a) for the buckled shape. Finally, Fig. 4 shows the energy release rate as a function of the delamination size θ_0 for specific levels of midpoint deflection. An initial delamination size of $\theta_0 = 5$ deg and a thickness of $h_l/h_s = 2/30$ were assumed. The energy release rate has been normalized with the quantity $E_2h_s/(1 - \nu_{12}\nu_{21})$. The rapidly dropping G with delamination size indicates that a growth criterion based on a critical energy release rate would more likely predict stable growth of the delamination for this particular geometry.

Transverse shear effects can be assessed on the basis of Eq. (24) for the critical pressure. It is seen that the effect of transverse shearing forces is increased for smaller delamination ranges θ_0 as well as for larger delamination thicknesses h_l. For our example case, with moduli ratio $E_2/G_{12} = 10$, shell thickness $h_s/R_0 = 0.05$, delamination thickness $h_l/h_s = 4/30$, and delamination range $\theta_0 = 15$ deg, Eq. (24) would predict a reduction in the delamination buckling load p_{cr} of 0.54% (vs the corresponding values without transverse shear effects).

On a $\theta_0 = 5$ deg delamination of thickness $h_l/h_s = 1/30$ the critical load would be 0.30% lower, whereas for the same delamination thickness $h_l/h_s = 1/30$ but larger size of $\theta_0 = 10$ deg, transverse shearing forces would reduce the buckling load by only 0.08%. Since we are considering a thin shell, these effects are, in general, minor. Transverse shear effects are expected to be more important for thicker delaminations that would normally arise in thick shells. Transverse shear effects are also expected to increase the energy release rate since the additional energy due to the transverse shearing forces would be included.[3]

Conclusions

In summary, an approximate analytical model was developed to study the stability of composite shells with thin delaminations under external pressure. Thin film approximation is based on regarding the unbuckled portion of the structure as being "infinitely thick" with respect to the delaminated layer. The quantitative aspects of the problem were investigated over a range of values for the delamination size and thickness. Closed-form expressions for the critical pressure [Eq. (7b)] and growth characteristics [Eq. (14) for the energy release rate] are derived, as well as for the cutoff level of the delamination range below which local delamination buckling cannot take place [Eq. (26)]. Furthermore, an investigation of the transverse shear effects on both the buckling load and the growth characteristics was performed. These effects are found to cause a reduction in the critical load [Eq. (24) for the critical pressure with transverse shear effects].

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